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(α, ψ, ξ)-contractive multivalued mappings

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Abstract

In this paper, we introduce the notion of (α, ψ, ξ)-contractive multivalued mappings to generalize and extend the notion of α-ψ-contractive mappings to closed valued multifunctions. We investigate the existence of fixed points for such mappings. We also construct an example to show that our result is more general than the results of α-ψ-contractive closed valued multifunctions.

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1 Introduction and preliminaries

Recently, Samet *et al.* [1] introduced the notions of α-ψ-contractive and α-admissible self-mappings and proved some fixed-point results for such mappings in complete metric spaces. Karapınar and Samet [2] generalized these notions and obtained some fixed-point results. Asl *et al.* [3] extended these notions to multifunctions by introducing the notions of α_*-ψ-contractive and α_*-admissible mappings and proved some fixed-point results. Some results in this direction are also given in [4–6]. Ali and Kamran [7] further generalized the notion of α_*-ψ-contractive mappings and obtained some fixed-point theorems for multivalued mappings. Salimi *et al.* [8] modified the notions of α-ψ-contractive and α-admissible self-mappings by introducing another function η and established some fixed-point theorems for such mappings in complete metric spaces. N. Hussain *et al.* [9] extended these modified notions to multivalued mappings. Recently, Mohammadi and Rezapour [10] showed that the results obtained by Salimi *et al.* [8] follow from corresponding results for α-ψ-contractive mappings. More recently, Berzig and Karapinar [11] proved that the first main result of Salimi *et al.* [8] follows from a result of Karapınar and Samet [2]. The purpose of this paper is to introduce the notion of (α, ψ, ξ)-contractive multivalued mappings to generalize and extend the notion of α-ψ-contractive mappings to closed valued multifunctions and to provide fixed-point theorems for (α, ψ, ξ)-contractive multivalued mappings in complete metric spaces.

We recollect the following definitions, for the sake of completeness. Let (X, d) be a metric space. We denote by $CB(X)$ the class of all nonempty closed and bounded subsets of X and by $CL(X)$ the class of all nonempty closed subsets of X. For every $A, B \in CL(X)$, let

$$H(A,B) = \begin{cases} \max\{\sup_{x\in A} d(x,B), \sup_{y\in B} d(y,A)\}, & \text{if the maximum exists;} \\ \infty, & \text{otherwise.} \end{cases}$$

Such a map H is called the generalized Hausdorff metric induced by the metric d. Let Ψ be a set of nondecreasing functions, $\psi : [0,\infty) \to [0,\infty)$ such that $\sum_{n=1}^{\infty} \psi^n(t) < \infty$ for each $t > 0$, where ψ^n is the nth iterate of ψ. It is known that for each $\psi \in \Psi$, we have $\psi(t) < t$ for all $t > 0$ and $\psi(0) = 0$ for $t = 0$. More details as regards such a function can be found in *e.g.* [1, 2].

Definition 1.1 [3] Let (X,d) be a metric space and $\alpha : X \times X \to [0,\infty)$ be a mapping. A mapping $G : X \to CL(X)$ is α_*-admissible if

$$\alpha(x,y) \geq 1 \quad \Rightarrow \quad \alpha_*(Gx,Gy) \geq 1,$$

where $\alpha_*(Gx,Gy) = \inf\{\alpha(a,b) : a \in Gx, b \in Gy\}$.

2 Main results

We begin this section by considering a family Ξ of functions $\xi : [0,\infty) \to [0,\infty)$ satisfying the following conditions:

(i) ξ is continuous;
(ii) ξ is nondecreasing on $[0,\infty)$;
(iii) $\xi(0) = 0$ and $\xi(t) > 0$ for all $t \in (0,\infty)$;
(iv) ξ is subadditive.

Example 2.1 Suppose that $\phi : [0,\infty) \to [0,\infty)$ is a Lebesgue integrable mapping which is summable on each compact subset of $[0,\infty)$, for each $\epsilon > 0$, $\int_0^\epsilon \phi(t)\,dt > 0$, and for each $a, b > 0$, we have

$$\int_0^{a+b} \phi(t)\,dt \leq \int_0^a \phi(t)\,dt + \int_0^b \phi(t)\,dt.$$

Define $\xi : [0,\infty) \to [0,\infty)$ by $\xi(t) = \int_0^t \phi(w)\,dw$ for each $t \in [0,\infty)$. Then $\xi \in \Xi$.

Lemma 2.2 *Let (X,d) is a metric space and let $\xi \in \Xi$. Then $(X, \xi \circ d)$ is a metric space.*

Lemma 2.3 *Let (X,d) be a metric space, let $\xi \in \Xi$ and let $B \in CL(X)$. Assume that there exists $x \in X$ such that $\xi(d(x,B)) > 0$. Then there exists $y \in B$ such that*

$$\xi\big(d(x,y)\big) < q\xi\big(d(x,B)\big),$$

where $q > 1$.

Proof By hypothesis we have $\xi(d(x,B)) > 0$. We choose

$$\epsilon = (q-1)\xi\big(d(x,B)\big).$$

By the definition of an infimum, since $\xi \circ d$ is a metric space, it follows that there exists $y \in B$ such that

$$\xi\big(d(x,y)\big) < \xi\big(d(x,B)\big) + \epsilon = q\xi\big(d(x,B)\big). \qquad \square$$

Definition 2.4 Let (X,d) be a metric space. A mapping $G : X \to CL(X)$ is called (α,ψ,ξ)-contractive if there exist three functions $\psi \in \Psi$, $\xi \in \Xi$ and $\alpha : X \times X \to [0,\infty)$ such that

$$x,y \in X, \quad \alpha(x,y) \geq 1 \quad \Rightarrow \quad \xi\big(H(Gx,Gy)\big) \leq \psi\big(\xi\big(M(x,y)\big)\big), \tag{2.1}$$

where $M(x,y) = \max\{d(x,y), d(x,Gx), d(y,Gy), \frac{d(x,Gy)+d(y,Gx)}{2}\}$.

In case when $\psi \in \Psi$ is strictly increasing, the (α,ψ,ξ)-contractive mapping is called a strictly (α,ψ,ξ)-contractive mapping.

Theorem 2.5 *Let (X,d) be a complete metric space and let $G : X \to CL(X)$ be a strictly (α,ψ,ξ)-contractive mapping satisfying the following assumptions*:
- (i) *G is an α_*-admissible mapping*;
- (ii) *there exist $x_0 \in X$ and $x_1 \in Gx_0$ such that $\alpha(x_0,x_1) \geq 1$*;
- (iii) *G is continuous.*

Then G has a fixed point.

Proof By hypothesis, there exist $x_0 \in X$ and $x_1 \in Gx_0$ such that $\alpha(x_0,x_1) \geq 1$. If $x_0 = x_1$, then we have nothing to prove. Let $x_0 \neq x_1$. If $x_1 \in Gx_1$, then x_1 is a fixed point. Let $x_1 \notin Gx_1$. Then from equation (2.1), we have

$$\begin{aligned} 0 &< \xi\big(H(Gx_0,Gx_1)\big) \\ &\leq \psi\left(\xi\left(\max\left\{d(x_0,x_1), d(x_0,Gx_0), d(x_1,Gx_1), \frac{d(x_0,Gx_1)+d(x_1,Gx_0)}{2}\right\}\right)\right) \\ &= \psi\big(\xi\big(\max\{d(x_0,x_1), d(x_1,Gx_1)\}\big)\big), \end{aligned} \tag{2.2}$$

since $\frac{d(x_0,Gx_1)}{2} \leq \max\{d(x_0,x_1), d(x_1,Gx_1)\}$. Assume that $\max\{d(x_0,x_1), d(x_1,Gx_1)\} = d(x_1, Gx_1)$. Then from equation (2.2), we have

$$\begin{aligned} 0 &< \xi\big(d(x_1,Gx_1)\big) \leq \xi\big(H(Gx_0,Gx_1)\big) \\ &\leq \psi\big(\xi\big(\max\{d(x_0,x_1), d(x_1,Gx_1)\}\big)\big) \\ &= \psi\big(\xi\big(d(x_1,Gx_1)\big)\big), \end{aligned} \tag{2.3}$$

which is a contradiction. Hence, $\max\{d(x_0,x_1), d(x_1,Gx_1)\} = d(x_0,x_1)$. Then from equation (2.2), we have

$$0 < \xi\big(d(x_1,Gx_1)\big) \leq \xi\big(H(Gx_0,Gx_1)\big) \leq \psi\big(\xi\big(d(x_0,x_1)\big)\big). \tag{2.4}$$

For $q > 1$ by Lemma 2.3, there exists $x_2 \in Gx_1$ such that

$$0 < \xi\big(d(x_1,x_2)\big) < q\xi\big(d(x_1,Gx_1)\big). \tag{2.5}$$

From equations (2.4) and (2.5), we have

$$0 < \xi\big(d(x_1,x_2)\big) < q\psi\big(\xi\big(d(x_0,x_1)\big)\big). \tag{2.6}$$

Applying ψ in equation (2.6), we have

$$0 < \psi\big(\xi\big(d(x_1,x_2)\big)\big) < \psi\big(q\psi\big(\xi\big(d(x_0,x_1)\big)\big)\big). \tag{2.7}$$

Put $q_1 = \frac{\psi(q\psi(\xi(d(x_0,x_1))))}{\psi(\xi(d(x_1,x_2)))}$. Then $q_1 > 1$. Since G is an α_*-admissible mapping, then $\alpha_*(Gx_0, Gx_1) \geq 1$. Thus we have $\alpha(x_1,x_2) \geq \alpha_*(Gx_0,Gx_1) \geq 1$. If $x_2 \in Gx_2$, then x_2 is a fixed point. Let $x_2 \notin Gx_2$. Then from equation (2.1), we have

$$\begin{aligned} 0 &< \xi\big(H(Gx_1,Gx_2)\big) \\ &\leq \psi\left(\xi\left(\max\left\{d(x_1,x_2), d(x_1,Gx_1), d(x_2,Gx_2), \frac{d(x_1,Gx_2)+d(x_2,Gx_1)}{2}\right\}\right)\right) \\ &= \psi\big(\xi\big(\max\big\{d(x_1,x_2), d(x_2,Gx_2)\big\}\big)\big), \end{aligned} \tag{2.8}$$

since $\frac{d(x_1,Gx_2)}{2} \leq \max\{d(x_1,x_2), d(x_2,Gx_2)\}$. Assume that $\max\{d(x_1,x_2), d(x_2,Gx_2)\} = d(x_2, Gx_2)$. Then from equation (2.8), we have

$$\begin{aligned} 0 &< \xi\big(d(x_2,Gx_2)\big) \leq \xi\big(H(Gx_1,Gx_2)\big) \\ &\leq \psi\big(\xi\big(\max\big\{d(x_1,x_2), d(x_2,Gx_2)\big\}\big)\big) \\ &= \psi\big(\xi\big(d(x_2,Gx_2)\big)\big), \end{aligned} \tag{2.9}$$

which is a contradiction. Hence, $\max\{d(x_1,x_2), d(x_2,Gx_2)\} = d(x_1,x_2)$. Then from equation (2.8), we have

$$0 < \xi\big(d(x_2,Gx_2)\big) \leq \xi\big(H(Gx_1,Gx_2)\big) \leq \psi\big(\xi\big(d(x_1,x_2)\big)\big). \tag{2.10}$$

For $q_1 > 1$ by Lemma 2.3, there exists $x_3 \in Gx_2$ such that

$$0 < \xi\big(d(x_2,x_3)\big) < q_1\xi\big(d(x_2,Gx_2)\big). \tag{2.11}$$

From equations (2.10) and (2.11), we have

$$0 < \xi\big(d(x_2,x_3)\big) < q_1\psi\big(\xi\big(d(x_1,x_2)\big)\big) = \psi\big(q\psi\big(\xi\big(d(x_0,x_1)\big)\big)\big). \tag{2.12}$$

Applying ψ in equation (2.12), we have

$$0 < \psi\big(\xi\big(d(x_2,x_3)\big)\big) < \psi^2\big(q\psi\big(\xi\big(d(x_0,x_1)\big)\big)\big). \tag{2.13}$$

Put $q_2 = \frac{\psi^2(q\psi(\xi(d(x_0,x_1))))}{\psi(\xi(d(x_2,x_3)))}$. Then $q_2 > 1$. Since G is an α_*-admissible mapping, then $\alpha_*(Gx_1, Gx_2) \geq 1$. Thus we have $\alpha(x_2,x_3) \geq \alpha_*(Gx_1,Gx_2) \geq 1$. If $x_3 \in Gx_3$, then x_3 is a fixed point. Let $x_3 \notin Gx_3$. Then from equation (2.1), we have

$$\begin{aligned} 0 &< \xi\big(H(Gx_2,Gx_3)\big) \\ &\leq \psi\left(\xi\left(\max\left\{d(x_2,x_3), d(x_2,Gx_2), d(x_3,Gx_3), \frac{d(x_2,Gx_3)+d(x_3,Gx_2)}{2}\right\}\right)\right) \\ &= \psi\big(\xi\big(\max\big\{d(x_2,x_3), d(x_3,Gx_3)\big\}\big)\big), \end{aligned} \tag{2.14}$$

since $\frac{d(x_2,Gx_3)}{2} \leq \max\{d(x_2,x_3), d(x_3,Gx_3)\}$. Assume that $\max\{d(x_2,x_3), d(x_3,Gx_3)\} = d(x_3, Gx_3)$. Then from equation (2.14), we have

$$\begin{aligned} 0 &< \xi\big(d(x_3,Gx_3)\big) \leq \xi\big(H(Gx_2,Gx_3)\big) \\ &\leq \psi\big(\xi\big(\max\big\{d(x_2,x_3), d(x_3,Gx_3)\big\}\big)\big) \\ &= \psi\big(\xi\big(d(x_3,Gx_3)\big)\big), \end{aligned} \tag{2.15}$$

which is a contradiction to our assumption. Hence, $\max\{d(x_2,x_3), d(x_3,Gx_3)\} = d(x_2,x_3)$. Then from equation (2.14), we have

$$0 < \xi\big(d(x_3,Gx_3)\big) \leq \xi\big(H(Gx_2,Gx_3)\big) \leq \psi\big(\xi\big(d(x_2,x_3)\big)\big). \tag{2.16}$$

For $q_2 > 1$ by Lemma 2.3, there exists $x_4 \in Gx_3$ such that

$$0 < \xi\big(d(x_3,x_4)\big) < q_2\xi\big(d(x_3,Gx_3)\big). \tag{2.17}$$

From equations (2.16) and (2.17), we have

$$0 < \xi\big(d(x_3,x_4)\big) < q_2\psi\big(\xi\big(d(x_2,x_3)\big)\big) = \psi^2\big(q\psi\big(\xi\big(d(x_0,x_1)\big)\big)\big). \tag{2.18}$$

Continuing the same process, we get a sequence $\{x_n\}$ in X such that $x_{n+1} \in Gx_n$, $x_{n+1} \neq x_n$, $\alpha(x_n,x_{n+1}) \geq 1$, and

$$\xi\big(d(x_{n+1},x_{n+2})\big) < \psi^n\big(q\psi\big(\xi\big(d(x_0,x_1)\big)\big)\big) \quad \text{for each } n \in \mathbb{N} \cup \{0\}. \tag{2.19}$$

Let $m > n$, we have

$$\xi\big(d(x_m,x_n)\big) \leq \sum_{i=n}^{m-1} \xi\big(d(x_i,x_{i+1})\big) < \sum_{i=n}^{m-1} \psi^{i-1}\big(q\psi\big(\xi\big(d(x_0,x_1)\big)\big)\big).$$

Since $\psi \in \Psi$, we have

$$\lim_{n,m\to\infty} \xi\big(d(x_m,x_n)\big) = 0. \tag{2.20}$$

This implies that

$$\lim_{n,m\to\infty} d(x_m,x_n) = 0. \tag{2.21}$$

Hence $\{x_n\}$ is a Cauchy sequence in (X,d). By completeness of (X,d), there exists $x^* \in X$ such that $x_n \to x^*$ as $n \to \infty$. Since G is continuous, we have

$$d\big(x^*, Gx^*\big) = \lim_{n\to\infty} d\big(x_{n+1}, Gx^*\big) \leq \lim_{n\to\infty} H\big(Gx_n, Gx^*\big) = 0.$$

Thus $x^* = Gx^*$. □

Theorem 2.6 *Let (X,d) be a complete metric space and let $G : X \to CL(X)$ be a strictly (α,ψ,ξ)-contractive mapping satisfying the following assumptions*:

(i) *G is an α_*-admissible mapping*;

(ii) *there exist $x_0 \in X$ and $x_1 \in Gx_0$ such that $\alpha(x_0,x_1) \geq 1$*;

(iii) *if $\{x_n\}$ is a sequence in X with $x_n \to x$ as $n \to \infty$ and $\alpha(x_n,x_{n+1}) \geq 1$ for each $n \in \mathbb{N} \cup \{0\}$, then we have $\alpha(x_n,x) \geq 1$ for each $n \in \mathbb{N} \cup \{0\}$.*

Then G has a fixed point.

Proof Following the proof of Theorem 2.5, we know that $\{x_n\}$ is a Cauchy sequence in X with $x_n \to x^*$ as $n \to \infty$ and $\alpha(x_n,x_{n+1}) \geq 1$ for each $n \in \mathbb{N} \cup \{0\}$. By hypothesis (iii), we have $\alpha(x_n,x^*) \geq 1$ for each $n \in \mathbb{N} \cup \{0\}$. Then from equation (2.1), we have

$$\begin{aligned}\xi\bigl(H(Gx_n,Gx^*)\bigr) \leq \psi\biggl(\xi\biggl(\max\biggl\{&d(x_n,x^*),d(x_n,Gx_n),d(x^*,Gx^*),\\&\frac{d(x_n,Gx^*)+d(x^*,Gx_n)}{2}\biggr\}\biggr)\biggr).\end{aligned} \tag{2.22}$$

Suppose that $d(x^*,Gx^*) \neq 0$.

We let $x_n \to x^*$. Taking $\epsilon = \frac{d(x^*,Gx^*)}{2}$ we can find $N_1 \in \mathbb{N}$ such that

$$d(x^*,x_m) < \frac{d(x^*,Gx^*)}{2} \quad \text{for each } m \geq N_1. \tag{2.23}$$

Moreover, as $\{x_n\}$ is a Cauchy sequence, there exists $N_2 \in \mathbb{N}$ such that

$$d(x_m,Gx_m) \leq d(x_m,x_{m+1}) < \frac{d(x^*,Gx^*)}{2} \quad \text{for each } m \geq N_2. \tag{2.24}$$

Furthermore,

$$d(x^*,Gx_m) \leq d(x^*,x_{m+1}) < \frac{d(x^*,Gx^*)}{2} \quad \text{for each } m \geq N_1. \tag{2.25}$$

As $d(x_m,Gx^*) \to d(x^*,Gx^*)$. Taking $\epsilon = \frac{d(x^*,Gx^*)}{2}$ we can find $N_3 \in \mathbb{N}$ such that

$$d(x_m,Gx^*) < \frac{3d(x^*,Gx^*)}{2} \quad \text{for each } m \geq N_3. \tag{2.26}$$

It follows from equations (2.23), (2.24), (2.25), and (2.26) that

$$\begin{aligned}&\max\biggl\{d(x_m,x^*),d(x_m,Gx_m),d(x^*,Gx^*),\frac{d(x_m,Gx^*)+d(x^*,Gx_m)}{2}\biggr\}\\&\quad= d(x^*,Gx^*),\end{aligned}$$

for $m \geq N = \max\{N_1,N_2,N_3\}$. Moreover, for $m \geq N$, by the triangle inequality, we have

$$\begin{aligned}\xi\bigl(d(x^*,Gx^*)\bigr) &\leq \xi\bigl(d(x^*,x_{m+1})\bigr)+\xi\bigl(H(Gx_m,Gx^*)\bigr)\\&\leq \xi\bigl(d(x^*,x_{m+1})\bigr)+\psi\biggl(\xi\biggl(\max\biggl\{d(x_m,x^*),d(x_m,Gx_m),d(x^*,Gx^*),\end{aligned}$$

$$\left.\frac{d(x_m, Gx^*) + d(x^*, Gx_m)}{2}\right\}\Bigg)\Bigg)$$

$$= \xi\big(d(x^*, x_{m+1})\big) + \psi\big(\xi\big(d(x^*, Gx^*)\big)\big). \tag{2.27}$$

Letting $m \to \infty$ in the above inequality, we have

$$\xi\big(d(x^*, Gx^*)\big) \leq \psi\big(\xi\big(d(x^*, Gx^*)\big)\big). \tag{2.28}$$

This is not possible if $\xi(d(x^*, Gx^*)) > 0$. Therefore, we have $\xi(d(x^*, Gx^*)) = 0$, which implies that $d(x^*, Gx^*) = 0$, *i.e.*, $x^* = Gx^*$. □

Example 2.7 Let $X = \mathbb{R}$ be endowed with the usual metric d. Define $G : X \to CL(X)$ by

$$Gx = \begin{cases} (-\infty, 0] & \text{if } x < 0, \\ \{0, \frac{x}{4}\} & \text{if } 0 \leq x < 2, \\ \{0\} & \text{if } x = 2, \\ [x^2, \infty) & \text{if } x > 2 \end{cases}$$

and $\alpha : X \times X \to [0, \infty)$ by

$$\alpha(x, y) = \begin{cases} 1 & \text{if } x, y \in [0, 2], \\ \frac{1}{2} & \text{otherwise.} \end{cases}$$

Take $\psi(t) = \frac{t}{2}$ and $\xi(t) = \sqrt{t}$ for each $t \geq 0$. Then G is an (α, ψ, ξ)-contractive mapping. For $x_0 = 1$ and $0 \in Gx_0$ we have $\alpha(1, 0) = 1$. Also, for each $x, y \in X$ with $\alpha(x, y) = 1$, we have $\alpha_*(Gx, Gy) = 1$. Moreover, for any sequence $\{x_n\}$ in X with $x_n \to x$ as $n \to \infty$ and $\alpha(x_n, x_{n+1}) = 1$ for each $n \in \mathbb{N} \cup \{0\}$, we have $\alpha(x_n, x) = 1$ for each $n \in \mathbb{N} \cup \{0\}$. Therefore, all conditions of Theorem 2.6 are satisfied and G has infinitely many fixed points. Note that Nadler's fixed-point theorem is not applicable here; see, for example, $x = 1.5$ and $y = 2$.

3 Consequences

It can be seen, by restricting $G : X \to X$ and taking $\xi(t) = t$ for each $t \geq 0$ in Theorems 2.5 and 2.6, that:

- Theorem 2.1 and Theorem 2.2 of Samet *et al.* [1] are special cases of Theorem 2.5 and Theorem 2.6, respectively;
- Theorem 2.3 of Asl *et al.* [3] is a special case of Theorem 2.6;
- Theorem 2.1 of Amiri *et al.* [5] is a special case of Theorem 2.5;
- Theorem 2.1 of Salimi *et al.* [8] is a special case of Theorems 2.5 and 2.6.

Further, it can be seen, by considering $G : X \to CB(X)$ and $\xi(t) = t$ for each $t \geq 0$, that

- Theorem 3.1 and Theorem 3.4 of Mohammadi *et al.* [4] are special cases of our results;
- Theorem 2.2 of Amiri *et al.* [5] is a special case of Theorem 2.6, when $\psi \in \Psi$ is sublinear.

Remark 3.1 Observe that, in case $G : X \to X$, ψ may be a nondecreasing function in Theorem 2.5 and Theorem 2.6.

Remark 3.2 Note that in Example 2.7, $\xi(t) = \sqrt{t}$. Therefore, one may not apply the aforementioned results and, as a consequence, conclude that G has a fixed point.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally in writing this article. All authors read and approved the final manuscript.

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References

1. Samet, B, Vetro, C, Vetro, P: Fixed point theorems for α-ψ-contractive type mappings. Nonlinear Anal. **75**, 2154-2165 (2012)
2. Karapinar, E, Samet, B: Generalized α-ψ-contractive type mappings and related fixed point theorems with applications. Abstr. Appl. Anal. **2012**, Article ID 793486 (2012)
3. Asl, JH, Rezapour, S, Shahzad, N: On fixed points of α-ψ-contractive multifunctions. Fixed Point Theory Appl. **2012**, Article ID 212 (2012). doi:10.1186/1687-1812-2012-212
4. Mohammadi, B, Rezapour, S, Shahzad, N: Some results on fixed points of α-ψ-Ciric generalized multifunctions. Fixed Point Theory Appl. **2013**, Article ID 24 (2013). doi:10.1186/1687-1812-2013-24
5. Amiri, P, Rezapour, S, Shahzad, N: Fixed points of generalized α-ψ-contractions. Rev. R. Acad. Cienc. Exactas Fís. Nat., Ser. A Mat. (2013). doi:10.1007/s13398-013-0123-9
6. Minak, G, Altun, I: Some new generalizations of Mizoguchi-Takahashi type fixed point theorem. J. Inequal. Appl. **2013**, Article ID 493 (2013). doi:10.1186/1029-242X-2013-493
7. Ali, MU, Kamran, T: On (α^*, ψ)-contractive multi-valued mappings. Fixed Point Theory Appl. **2013**, Article ID 137 (2013). doi:10.1186/1687-1812-2013-137
8. Salimi, P, Latif, A, Hussain, N: Modified α-ψ-contractive mappings with applications. Fixed Point Theory Appl. **2013**, Article ID 151 (2013). doi:10.1186/1687-1812-2013-151
9. Hussain, N, Salimi, P, Latif, A: Fixed point results for single and set-valued α-η-ψ-contractive mappings. Fixed Point Theory Appl. **2013**, Article ID 212 (2013). doi:10.1186/1687-1812-2013-212
10. Mohammadi, B, Rezapour, S: On modified α-φ-contractions. J. Adv. Math. Stud. **6**, 162-166 (2013)
11. Berzig, M, Karapinar, E: Note on 'Modified α-ψ-contractive mappings with application'. Thai J. Math. (in press)

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